Exam. Code : 211002
Subject Code : 4275

M.Sc. (Mathematics) 2nd Semester

MATH-562 : TENSORS AND DIFFERENTIAL GEOMETRY

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt ***five*** questions in all, selecting at least ***one*** from each section. All questions carry equal marks.

SECTION—A

1. (a) Define Cartesian tensor of order three. Also state and prove quotation law of tensors.

   (b) Define substitution tensor δ_{ij} and show that it is a tensor of order two.

2. (a) Show that :

   (i) The Christoffel symbols [ij, k] and $\left\{ \begin{matrix} k \\ ij \end{matrix} \right\}$ are symmetric in i and j.

   (ii) $[ij, k] = g_{kh} \left\{ \begin{matrix} h \\ ij \end{matrix} \right\}$, where symbols have their usual meaning.

   (b) Show that Christoffel symbols of first kind are not tensor quantities.

SECTION—B

3. (a) State and prove Serret Frenet formulae.

(b) Show that a necessary and sufficient condition that a curve lies on a sphere is that $\frac{\rho}{\sigma}+\frac{d}{ds}\left(\frac{\rho'}{\tau}\right)=0$ at every point on the curve.

4. (a) Show that in any cylindrical helix, the principal normal is normal to the cylinder, the binormal makes a constant angle with the axis and the ratio of curvature to torsion is constant.

(b) Define Bertrand curves. Show that the distance between corresponding points of two Bertrand curve is constant.

SECTION—C

5. (a) Find the envelope of the family of planes $3a^2x - 3ay + z = a^3$ and show that its edge of regression is the curve of intersection of the surfaces $y^2 = zx$, $xy = z$.

(b) Find the principal directions and principal curvatures on the surface $x = a(u + v)$, $y = b(u - v)$, $z = uv$.

6. (a) Define the asymptotic lines. Find the asymptotic lines on that catenoid of revolution $u = c \cosh\frac{z}{c}$.

(b) Show that when the lines of curvature are chosen on parametric curves, the Codazzi relations expressed in terms of E, F, G, L, N and their derivatives are $L_2 = \frac{E_2}{2}\left(\frac{L}{E}+\frac{N}{G}\right)$, $N_1 = \frac{G_1}{2}\left(\frac{L}{E}+\frac{N}{G}\right)$.

SECTION—D

7. (a) The necessary and sufficient condition that on the general surface, the curve $v = c$ be geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when $v = c$ for all values of u, where symbols have their usual meaning.

(b) Prove that the torsion of the geodesic tangent at any point of curve on a surface is given by $\frac{1}{\sigma} = \left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$ $\sin\theta \cos\theta$, where θ is the angle between the tangent and a principal direction and ρ_1 and ρ_2 are the principal radii of curve.

8. (a) State and prove Gauss-Bonnet theorem.

(b) State and prove Tissot's theorem.